Towards Parametrizing Logic Program Analysis: Two Examples

Lunjin Lu

Oakland University

Parametric Analysis

- A parametric analysis is an analysis whose input and output are parametrized with a number of parameters which can be instantiated to abstract properties after analysis is completed.
  - $\langle x \in \mathit{list}(\beta), qs(x,y)\rangle \leadsto \langle qs(x,y), y \in \mathit{list}(\beta)\rangle$
  - $\langle x \in \mathit{list}(nat), qs(x,y)\rangle \leadsto \langle qs(x,y), y \in \mathit{list}(nat)\rangle$
  - $\langle x \in \mathit{list}(int), qs(x,y)\rangle \leadsto \langle qs(x,y), y \in \mathit{list}(int)\rangle$.
  - ...

- Method for designing a parametric analysis by lifting a base analysis to Cousot's Cardinal Power domain.

Lifting Semantic Domains

Cardinal power $L_1^\sharp \xrightarrow{m} L_2^\sharp$ with base $L_2^\sharp$ and exponent $L_1^\sharp$ consists of all monotone functions from $L_1^\sharp$ to $L_2^\sharp$.

- $L_1^\sharp$ - abstract domain over which parameters range
- $L_2^\sharp$ - abstract domain of base analysis;
- (Cousot & Cousot) Let $\langle L_1, \alpha_1, L_1^\sharp, \gamma_1 \rangle$ and $\langle L_2, \alpha_2, L_2^\sharp, \gamma_2 \rangle$ be Galois connections. Then $\langle L_1 \xrightarrow{m} L_2, \alpha, L_1^\sharp \xrightarrow{m} L_2^\sharp, \gamma \rangle$ is a Galois connection where $\alpha = \lambda\phi.\alpha_2 \circ \phi \circ \gamma_1$ and $\gamma = \lambda\psi.\gamma_2 \circ \psi \circ \alpha_1$.

Lifting Semantic Functions

- Let $f : D^\sharp \xrightarrow{m} E^\sharp$. Define $\star_{L^\sharp} f : (L^\sharp \xrightarrow{m} D^\sharp) \xrightarrow{m} (L^\sharp \xrightarrow{m} E^\sharp)$ as

$$\star_{L^\sharp} f = \lambda\phi.(f \circ \phi)$$

- Lifting operators commute with composition, tupling and projection: for any $L^\sharp$,
  1. $\star_{L^\sharp}(f_2 \circ f_1) = (\star_{L^\sharp} f_2) \circ (\star_{L^\sharp} f_1)$,
  2. $\star_{L^\sharp}\langle f_1, f_2\rangle = \langle \star_{L^\sharp} f_1, \star_{L^\sharp} f_2\rangle$,
  3. $\star_{L^\sharp} proj_i(\langle \phi_1, \phi_2\rangle) = \phi_i$ for $i = 1, 2$, where $proj_i(\langle c_1, c_2\rangle) = c_i$ for $i = 1, 2$.

Properties of Parametric Analysis

- Parametric analysis + Instantiation of parameters = Instantiation of parameters + Base analysis;
- Correctness of the base analysis implies correctness of the parametric analysis;
- Optimality of the base analysis implies optimality of the parametric analysis.

Groundness dependency Analysis

- By Søndergaard and Marriott
- Tracks groundness dependencies between variables in a program state;
- Domain: $\langle \mathcal{P}os_U, \models \rangle$ of positive boolean formulae.

$$x_1 \wedge (x_2 \rightarrow x_3)$$

- Operations:
  - $\sqcup^{Gr} = \vee$.
  - $rename^{Gr}_{\vec{x} \mapsto \vec{y}}$ changes all occurrence of $\vec{x}$ to $\vec{y}$
  - $project^{Gr}_X(\phi) = \exists X.\phi$.
  - $ident^{Gr}_U = \mathbf{1}$.
  - $aunify^{Gr}_{x=t}(\phi) = \phi \wedge (x \leftrightarrow \bigwedge \mathbf{V}(t))$

Set Sharing Analysis

- By Codish, Søndergaard and Stuckey, based on an isomorphism between Jacobs and Langen's set sharing domain and $\mathcal{P}os$;
- Domain: $\langle \mathcal{P}os_U, \models \rangle$ of positive boolean formulae
- Operations:
  - $\sqcup^{Sh}$ same as $\sqcup^{Gr}$;
  - $rename^{Sh}$ same as $rename^{Gr}$;
  - $project^{Sh}$ same as $project^{Gr}$;
  - $ident^{Sh}_{U} = \bigwedge U \vee \bigvee_{x \in U} (\neg x \wedge \bigwedge (U \setminus \{x\}))$;
  - $aunify^{Sh}_{x=t}(\phi) = \phi \wedge (x \vee d) \vee H(\phi \wedge (\neg x \vee \neg d)) \wedge (\neg x \wedge \neg d)$ where $d = \bigwedge \mathbf{V}(t)$ and H takes a $\mathcal{P}os$ formula ψ to the smallest $\mathcal{D}ef$ formula ψ' such that $\psi \models \psi'$.

Parametric Groundness/Sharing Analyses

Exponent domain: $\mathcal{G}_{\mathbb{P}} = \{\wedge X \mid X \subseteq \mathbb{P}\}$ ordered by $\models$.

Cardinal Power: $\mathcal{G}_{\mathbb{P}} \overset{m}{\mapsto} \mathcal{P}os_U$

Abstract operations:

- $\phi_1\ (\star\sqcup^{\sharp})\ \phi_2 = \lambda g.(\phi_1(g)\sqcup^{\sharp}\phi_2(g))$,
- $\star project^{\sharp}_{X}(\phi) = project^{\sharp}_{X} \circ \phi$,
- $\star rename^{\sharp}_{\vec{x}\mapsto\vec{y}}(\phi) = rename^{\sharp}_{\vec{x}\mapsto\vec{y}} \circ \phi$,
- $\star ident^{\sharp}_{U} = \lambda g.ident^{\sharp}_{U}$, and
- $(\star aunify^{\sharp}_{x=t})(\phi) = aunify^{\sharp}_{x=t} \circ \phi$

Parametric Groundness Dependency

```
:- analysis(parametric,groundness,qs(X1,X2),true

app([],L,L).
app([X|L1],L2,[X|L3]) :-  app(L1,L2,L3).

pt([X|T],P,[X|B],A) :- leq(X,P), pt(T,P,B,A).
pt([X|T],P,B,[X|A]) :- gt(X,P),  pt(T,P,B,A).
pt([],_,[],[]).

leq(X,Y) :- X =< Y.
gt(X,Y) :- X > Y.

qs([],[]).
qs([X|Xs],Ys) :- pt(Xs,X,U,V), qs(U,S),
                 qs(V,L), app(S,[X|L],Ys).
```

Parametric Groundness Dependency

$$
\begin{array}{rcl}
\mathit{call\_gt}(x_1, x_2) & \text{:-} & \beta_1 \to x_1 x_2 \\
\mathit{ans\_gt}(x_1, x_2) & \text{:-} & x_1 x_2 \\
\mathit{call\_leq}(x_1, x_2) & \text{:-} & \beta_1 \to x_1 x_2 \\
\mathit{ans\_leq}(x_1, x_2) & \text{:-} & x_1 x_2 \\
\mathit{call\_pt}(x_1, x_2, x_3, x_4) & \text{:-} & \beta_1 \to x_1 x_2 \\
\mathit{ans\_pt}(x_1, x_2, x_3, x_4) & \text{:-} & (\beta_1 \to x_2) \wedge x_1 x_3 x_4 \\
\mathit{call\_qs}(x_1, x_2) & \text{:-} & (\beta_1 \to x_1) \wedge (\beta_2 \to (x_1 \vee x_2)) \\
\mathit{ans\_qs}(x_1, x_2) & \text{:-} & (x_1 \leftrightarrow x_2) \wedge ((\beta_1 \vee \beta_2) \to x_1 x_2) \\
\mathit{call\_app}(x_1, x_2, x_3) & \text{:-} & x_1 \wedge (\beta_1 \to x_2) \wedge (\beta_2 \to (x_2 \vee x_3)) \\
\mathit{ans\_app}(x_1, x_2, x_3) & \text{:-} & x_1 \wedge (x_2 \leftrightarrow x_3) \wedge ((\beta_1 \vee \beta_2) \to x_2 x_3)
\end{array}
$$

Parametric Set Sharing Analysis

```
:- analysis(parametric, sharing, qs(X1,X2), [[],[X1],[X2]]).
```

$$
\begin{array}{rcl}
call\_qs(x_1, x_2) & \text{:-} & x_1 \vee \neg\beta_1 x_2 \\
\beta = 1 & & \{\emptyset, \{x_2\}\} \\
\beta = 0 & & \{\emptyset, \{x_1\}, \{x_2\}\}
\end{array}
$$

Performance

- Experiments on a suite of 24 benchmarks of medium sizes.
- 2.33GHz Intel (R) Xeon (R) CPU, Linux 2.6.24 and SICSTUS Prolog 4.0.3 with CUDD 2.4.1.
- Abstract input: $(\beta_1 \rightarrow x_1) \wedge \cdots \wedge (\beta_n \rightarrow x_n) \wedge ident^{\sharp}_{\{x_1,\cdots,x_n\}}$
- Compare time of parametric analysis and average time of base analyses over 2^n instantiations.

	Groundness Ratio	Sharing Ratio
Minimum	0.88	0.99
Maximum	1.33	1.74
Average	1.023	1.087

Summary & Future Work

Summary

- Parametric analysis captures dependencies between analysis input and output;
- Proposed a method to parametrizing analysis by lifting to Cardinal power domain;
- Obtained parametric groudness and sharing analyses that are Pos-based;
- Generic analysis results are obtained with small performance penalty.

Future work

- Apply the method to other base and exponent domains, e.g., Reconstruct parametric type analysis;
- Resultant semantics versus Relational domains