Computing Loops with at Most One External Support Rule for Basic Logic Programs with Arbitrary Constraint Atoms

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Outline

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Motivations

- Previous work has shown that:
  - For normal and disjunctive programs, the well-founded models can be computed by unit propagation on program completion and loop formulas of loops with no external support.
  - When loop formulas of loops with exactly one external support are added, consequences beyond the well-founded model can be computed, which sometimes can significantly speed up answer set computation.
- We extend this approach to basic logic programs with abstract constraint atoms.

Basic Logic Programs

- An *abstract constraint atom (c-atom)* A is an expression of the form (D, C), where D is a finite set of atoms and $C \subseteq 2^D$. Specially, $A_d = D$ and $A_c = C$.
- A *basic logic program with c-atoms* (*logic program* or *program*) is a finite set of *rules* of the form

$$a \leftarrow A_1, \ldots, A_k, not\, A_{k+1}, \ldots, not\, A_n \tag{1}$$

  where a is an atom and A_i's are c-atoms.
- The notions of *r-answer set* and *c-answer set* are defined in (Son et al. JAIR 2007).

Loops and Loop Formulas

- *A compact representation of c-atoms*: Let S and J be two disjoint sets of atoms, $S \uplus J = \{S' \mid S \subseteq S' \text{ and } S' \subseteq S \cup J\}$. $S \uplus J$ is *maximal* in A if $S \uplus J \subseteq A_c$ and there is no other sets S' and J' s.t. $S' \uplus J' \subseteq A_c$ and $S \uplus J \subset S' \uplus J'$. A_c^* denotes the set of all maximal $S \uplus J$ in A
- The *dependency graph* G_P of a basic program P is a directed graph for atoms. (u, v) is a directed edge if there is a rule $r \in P$ such that $u = head(r)$ and $v \in S$, for some $S \uplus J \in A_c^*$ and $A \in body(r)$.
- A set L of atoms is called a *loop* of P, if L-induced subgraph of G_P is strongly connected.

Cont'd

- Let
  $\pi_A(L) = \bigvee_{S \uplus J \in A^*_{c|L}} S \wedge \neg(A_d \setminus (S \cup J))$,
  where $A^*_{c|L} = \{S \uplus J \in A^*_c \mid L \cap S = \emptyset\}$,
  $\sigma_A = \bigvee_{S \uplus J \in A^*_c} S \wedge \neg(A_d \setminus (S \cup J))$,
- For a basic rule r of the form (1), we define the formula
  $$\theta_L(r) = \pi_{A_1}(L) \wedge \cdots \wedge \pi_{A_k}(L) \wedge \neg\sigma_{A_{k+1}} \wedge \cdots \wedge \neg\sigma_{A_n}.$$
- A rule $r \in P$ is an *external support of* L if $head(r) \in L$ and $\theta_L(r) \not\equiv \bot$.
- The *loop formula* for L of P, denoted $LF_P(L)$, is defined as

$$\bigvee_{a \in L} a \supset \bigvee_{r \in R^-(L)} \theta_L(r)$$

where $R^-(L)$ is the set of external support rules of L.

Program Completion

- The *completion* of a basic program P, denoted by $Comp(P)$, consists of the following formulas:
  - $\bigwedge_{A \in pos(r)} \sigma_A \wedge \bigwedge_{A \in neg(r)} \neg\sigma_A \supset head(r)$, for each $r \in P$;
  - $a \supset \bigvee_{r \in P, head(r)=a} \left(\bigwedge_{A \in pos(r)} \sigma_A \wedge \bigwedge_{A \in neg(r)} \neg\sigma_A \right)$, for each $a \in \mathcal{A}$.

Theorem 1

Let P be a basic program. A set $M \subseteq \mathcal{A}$ is an r-answer set of P iff M is a model of $Comp(P) \cup \{LF_P(L) \mid L$ is a loop of $P\}$.

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Computing loop formulas of loops without external support

Function $ML_0(P, X, S)$

$ML := \emptyset$; $G :=$ the S induced subgraph of G_P;

For each strongly connected component L of G:

if $R^-(L, X) = \emptyset$ **then** add L to ML

else append $ML_0(P, X, L \setminus \{head(r) \mid r \in R^-(L, X)\})$ to ML.

return ML.

Let us define

$$loop_0(P, X) = \bigcup_{L \in ML_0(P, X, \mathcal{A})} \neg.L.$$

An iterative procedure

Input: A basic logic program P
Output: A set of literals X

1. let $X := \emptyset$, compute the completion of P and convert it to set of clauses C;
2. compute the loop formulas of the loops that have no external support rules under X, then add them to C;
3. apply unit propagation to C, append the computed consequences to X;
4. go back to step 2 until it does not produce any new consequences.

Properties

- The above procedure computes the well-founded model in [Wang et al. 2012].
- The procedure can be improved to compute the (ultimate) well-founded semantics of [Pelov et al. 2007] for aggregate programs with only monotonic aggregate atoms.
- If we add loop formulas of loops that have exactly one external support rule in the above procedure, it computes a super set of the well-founded model.

$$T^P(X) = UP(comp(P) \cup loop_0(P,X) \cup loop_1^u(P,X) \cup X).$$

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Some Experiments

- We implement the previous procedure for program P that can be accepted by lparse. It first computes the least fixpoint of the T^P operator, denoted $T(P)$, and then adds $\{\leftarrow not\, a \mid a \in T(P)\} \cup \{\leftarrow a \mid \neg a \in T(P)\}$ to P.
- We run our procedure on the familiar Hamiltonian Circuit (HC) problem encoded with the following cardinality constraints:

$$\leftarrow 2\{dhc(X,Y) : arc(X,Y)\}, vertex(Y).$$
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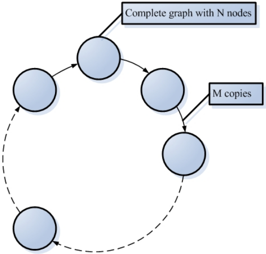
- The graphs have the structure:



Experiment Results

Problem	smodels	smodels$_T$	clasp	clasp$_T$	T
10x10	31.86	17.12	0.05	0.02	0.05
10x20	-	175.28	0.04	0.06	0.11
10x30	-	452.39	0.07	0.07	0.18
10x40	-	-	0.10	0.11	0.26
10x50	-	-	0.22	0.19	0.31
20x10	-	-	18.45	5.76	0.44
20x20	-	-	113.89	91.71	0.93
20x30	-	-	96.12	18.83	1.34
20x40	-	-	77.22	2.52	1.89
20x50	-	-	422.37	201.61	2.38

Table : Run-time Data for smodels and clasp.

- $M \times N$: a graph with N copies of the complete graph with M nodes.
- It shows that for most programs, information from $T(P)$ makes both smodels and clasp run faster, when lookahead operators are turned off.

Our contribution

- We extended the idea to basic logic programs with abstract constraint atoms.
- We gave algorithms for computing loop formulas of loops with at most one external support rule for basic logic programs.
- We considered how they, together with the program completion, can be used to deduce useful consequences of a logic program under unit propagation.
- We related the consequences computed here to the well-founded semantics in [Wang et al. 2012] and [Pelov et al. 2007].
- We showed that the loop formula approach for basic logic programs with c-atoms can also benefit answer set computation for some problems with a certain type of structure.

Thank you!

References

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