# ABA-Based Answer Set Justification

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- more specifically: **stable extension** semantics for argumentation frameworks
  $\Rightarrow$ common roots with stable model semantics
- Note: only **consistent extended logic programs** (no constraints, disjunction, ...)

# Answer Sets

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# Translated ABA framework

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Arguments: *label* : (*Assumptions*, *Facts*) $\vdash$ *conclusion*

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Corresponds to answer set $\{bird, wounded, \neg fly, abnormalBird\}$
$\Rightarrow$ every literal has a **corresponding argument**

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- supporting assumptions/facts of arguments in attack tree

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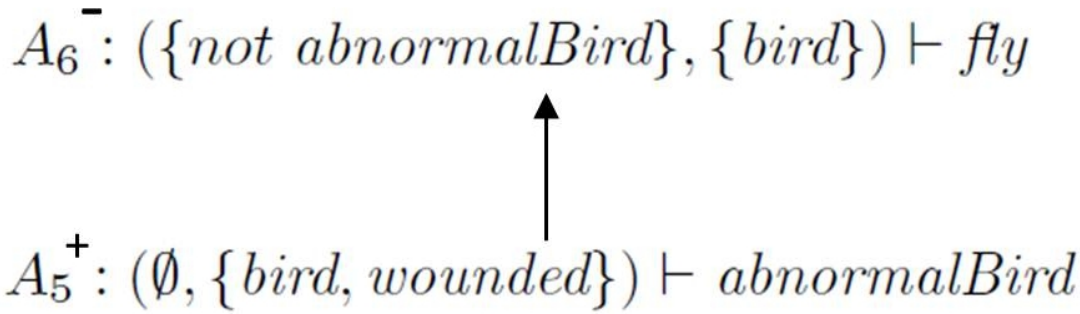

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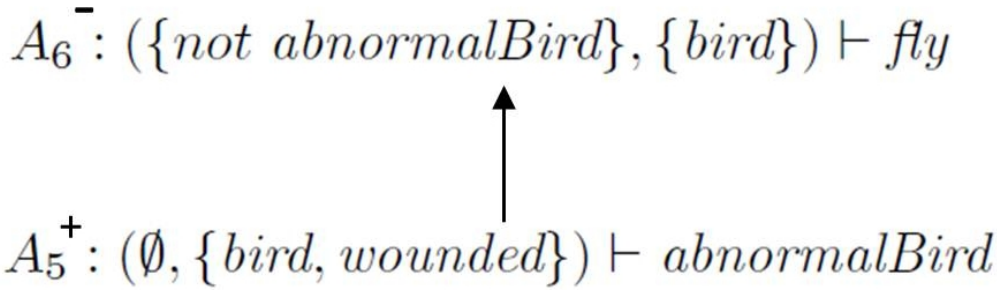


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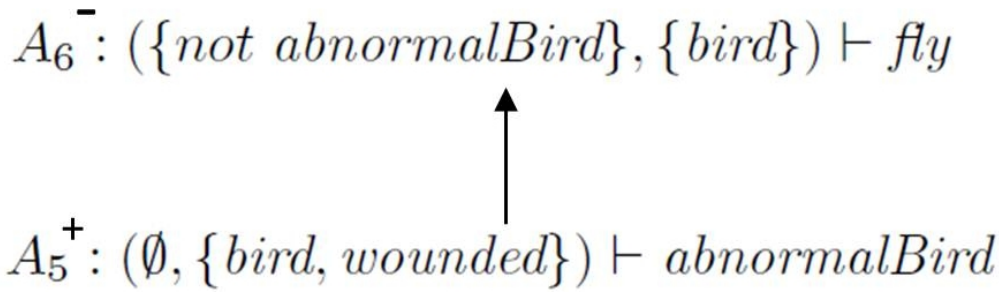


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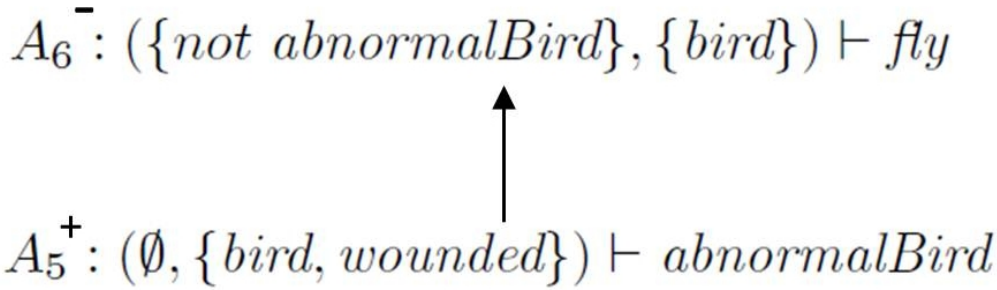


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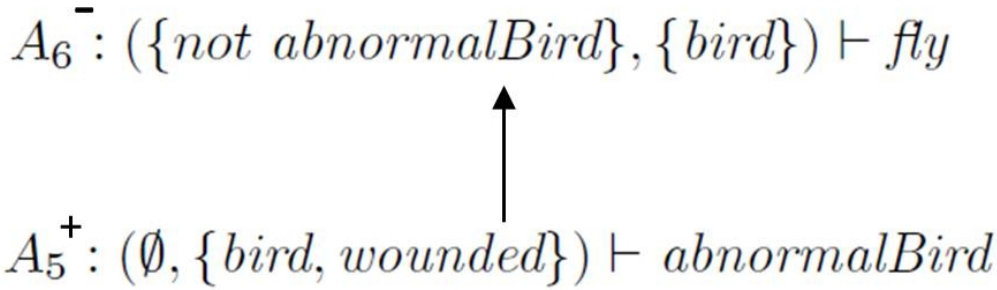


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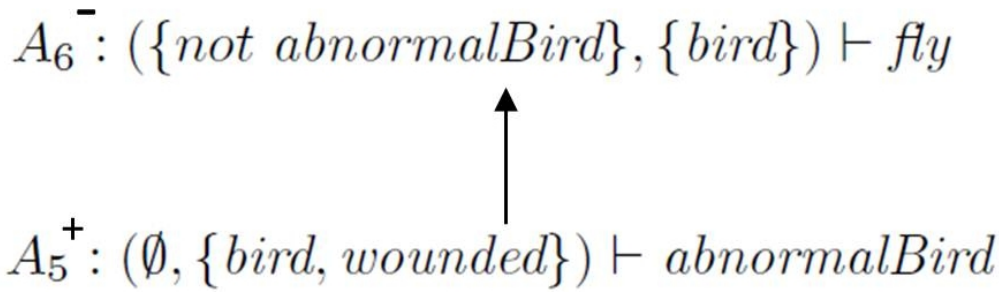


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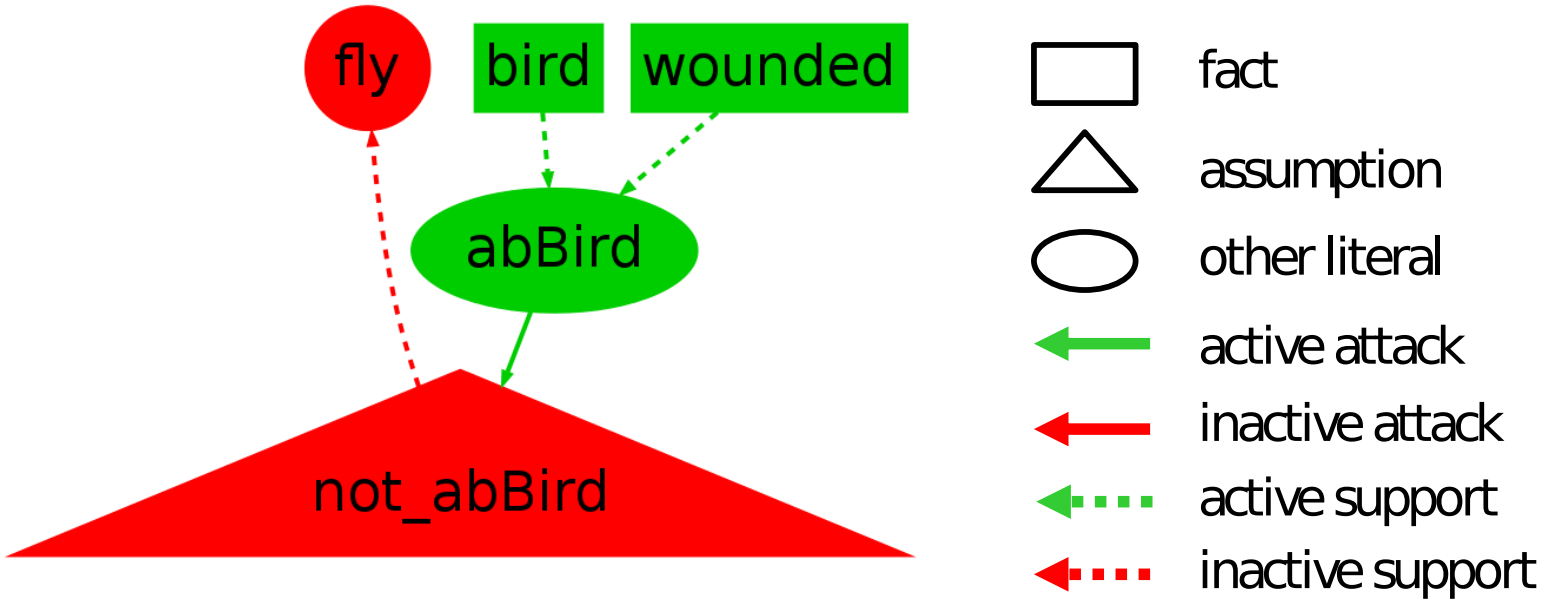

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Future work:

- justification for answer set programs with preferences
- possible applications: law, medicine

# Questions

Thank you for your attention!